Differentiating (1) with respect to x, we have

$$X - x = \frac{b^2}{a^2}x. \tag{2}$$

Solving (1) and (2) simultaneously, we obtain

$$X^2\left[1 - \frac{a^2}{a^2+b^2}\right]^2 + Y^2 = b^2 - \frac{b^2a^2}{(a^2+b^2)^2}X^2.$$

Reducing, we have

$$\frac{X^2}{a^2+b^2} + \frac{Y^2}{b^2} = 1.$$

This is an ellipse with same minor axis as original ellipse.

Also solved by Frank C. Moore, H. C. Feemster, and the Proposer.

MECHANICS.

281. Proposed by C. N. SCHMALL, New York City.

ABC is a triangle inscribed in a circle, center O, and L, M, N, are the centers of gravity of the sectors AOB, BOC, COA. Show that

$$\frac{AB}{OL} + \frac{BC}{OM} + \frac{CA}{ON} = 3\pi.$$

Solution by S. W. Reaves, University of Oklahoma.

The well-known formula for the center of gravity of a sector of a circle gives

$$OL = \frac{4}{3}\cdot\frac{r\sin\frac{1}{2}AOB}{\text{angle } AOB} = \frac{AB}{3\angle C}.$$

Hence $\dfrac{AB}{OL} = 3\angle C$. Similarly, $\dfrac{BC}{OM} = 3\angle A$, and $\dfrac{CA}{ON} = 3\angle B$. Adding,

$$\frac{AB}{OL} + \frac{BC}{OM} + \frac{CA}{ON} = 3(\angle A + \angle B + \angle C) = 3\pi.$$

Also solved by A. M. Harding, Charles E. Horne, P. Peñalver, B. Libby, Elmer Schuyler, Walter C. Eells, Richard Morris, H. C. Feemster, J. B. Smith, J. W. Colson, F. C. Reisler, and I. A. Barret.

282. Proposed by R. P. LOCHNER, Philadelphia, Pa.

A car weighing 10 tons (2,240 lbs. each) attains a speed of 15 miles an hour from rest in 24 seconds, during which it covers 100 yards. If the space-average of the resistances is 30 lbs. per ton, find the average horse-power used to drive the car. (Morley's *Mechanics for Engineers*, p. 66.)

Solution by J. W. Clawson, Collegeville, Pa.

Force (lbs.) required if there were no friction $= \dfrac{m}{g}\cdot\dfrac{v}{t} = \dfrac{2240}{32}\times\dfrac{22}{24} = 641$ lbs. approximately.

Force (lbs.) required to overcome friction = 300 lbs. Total force acting is therefore 941 lbs.

$$\text{Power applied} = \frac{Fs}{t} = \frac{941 \times 300}{24} = 11{,}762 \text{ ft. lbs. per sec.} = 21 \text{ H.P. (approx.)}.$$

283. Proposed by C. N. SCHMALL, New York City.

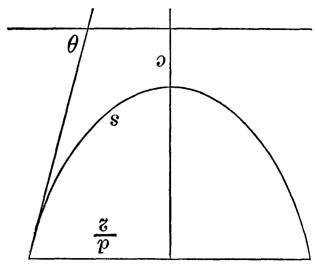
The maximum length of a certain chain which can be suspended from one end without breaking is l. It is desired to form a catenary with a length $2l/k$ of the chain, the points of support being a distance d apart, in the same horizontal line. Show that the maximum value of d is

$$\frac{2l}{k}(k^2-1)^{1/2}\log\left(\frac{k+1}{k-1}\right)^{1/2}.$$

SOLUTION BY A. M. HARDING, University of Arkansas.

Let w = weight per unit length. Then wl = maximum tension the chain will stand.

The tension at the point of support is given by $T\sin\theta = ws$ where s = one-half the length of the chain and θ is the angle that the tangent to the catenary at that point makes with the x-axis.



If we put $T = wl$ and $s = l/k$ we find $\sin\theta = 1/k$. But $\tan\theta = \frac{1}{2}(e^{d/2c} - e^{-(d/2c)})$ where c is the distance along the Y-axis from the origin to the catenary.

Hence we have

$$\tfrac{1}{2}(e^{d/2c} - e^{-(d/2c)}) = \frac{1}{\sqrt{k^2-1}}.$$

From this equation we obtain

$$d = 2c\log\left(\frac{k+1}{k-1}\right)^{\frac{1}{2}}.$$

We have also the intrinsic equation of the catenary $s = c\tan\theta$, from which we obtain

$$c = \frac{l}{k}\sqrt{k^2-1}.$$

Whence

$$d = \frac{2l}{k}(k^2-1)^{\frac{1}{2}}\log\left(\frac{k+1}{k-1}\right)^{\frac{1}{2}}.$$

Also solved by J. W. COLSON.

NUMBER THEORY.

189. Proposed by V. M. SPUNAR, Chicago, Illinois.

If p and $p_1 = 2^p - 1$ are primes, then are the numbers of the sequence $p_1 = 2^p - 1$, $p_2 = 2^{p_1} - 1$, $p_3 = 2^{p_2} - 1$, $\cdots$, $p_n = 2^{p_{n-1}} - 1$ all primes?